Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl16

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Version of record first published: 20 Apr 2011.

To cite this article: G. Barbero & A. Strigazzi (1982): On the Capacity Behaviour of a Nematic Liquid Crystal Cell in an Acoustic Re-Orienting Field, Molecular Crystals and Liquid Crystals, 82:1, 5-11

To link to this article: http://dx.doi.org/10.1080/01406568208070505

Mol. Cryst. Liq. Cryst. Vol. 82 (Letters), pp. 5-11
0140-6566/82/8201-0005$06.50/0

ON THE CAPACITY BEHAVIOUR OF A NEMATIC LIQUID CRYSTAL CELL IN AN ACOUSTIC RE-ORIENTING FIELD(*)

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(Submitted for publication January 27, 1982)

Abstract: The capacity dependence of a nematic liquid crystal cell on the ultrasound reduced coherence length Ξ is investigated, in the strong anchoring hypothesis. The calculation is performed in the frame of a static reorienting theory carried out by Dion and Jacob, concerning the ultrasound - nematic interaction. The obtained simple relations suggest an experimental method able to test the validity of such a theory.

Many efforts have been made during the last years, to improve acoustic imaging by means of nematic liquid crystals (NLC)[(1 ÷ 3)]. Hence the understanding of the ultrasound (US) - NLC interaction is essential both from a fundamental and from a practical point of view. Recently Dion and Jacob proposed a static theory of direct interaction[(4)], as a consequence of the theorem of minimum entropy production[(5)],

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(*) Research partially supported by Centro Ricerche FIAT S.p.A.

and that theory has been supported by some experimental evidence[6,7]. The aesthetical validity of this approach consists in the possibility of describing whatever external field effect always in terms of reorientational Freedericksz transitions (MCB, ECB and ACB), assuming a coherence length ξ, depending on the elastic energy and on the field anisotropic energy packed in the NLC.

In the frame of the isotropic elasticity,

$$(1) \qquad \xi^2 = K v / 2 h I \alpha_a$$

is assumed in the present case[4], and the tilt angle φ in the cell is given by:

$$(2) \qquad d^2 \varphi / d \eta^2 + \Xi^{-2} \sin 2 (\varphi - \beta) = 0$$

with the boundary condition φ ($\pm$ 1/2) = 0, in the strong anchoring hypothesis.

Now, according with the fact that the capacitance measurement is a useful method to describe the director lines distribution in a non dissipative NLC cell[8 ÷ 11], the authors purpose is to give an approximated solution φ (η) of eq. (2) in the case of $\beta \neq 0$, and a very simple approximated relation $\delta C / C_{\parallel}$ *vs.* Ξ^{-2}, valid if $\Xi > 1$, *i.e.* in the interesting situation in which the US intensity I is very low, and the streaming could be neglected(**). By following the perturbation method[12], and assuming Ξ^{-2} as expansion parameter, we have the uniformly conver-

(**) In the case of common NLC, $K \sim 10^{-6}$ dyn, $v \sim 1.5 \cdot 10^5$ cm/s, and $\alpha_a \sim$ $\sim 10^{-1}$ Nep/cm; hence for a cell having $h \sim 50\,\mu m$ irradiated by a US intensity $I \sim 10$ mW/cm^2 it results $\Xi^2 \sim 60$.

gent series(•):

$$(3) \qquad \varphi(\eta) = \sum_{k=0}^{\infty} \varphi_k \cdot \Xi^{-2k} \quad ,$$

and consequently:

$$(4) \qquad \sin 2(\varphi - \beta) = \sum_{n=0}^{\infty} q_n \cdot \Xi^{-2n} \quad ,$$

where the expansion coefficients are to be determined by means of the recurrence relations:

$$(5) \qquad \begin{cases} q_n = \sum_{k=0}^{\infty} (-1)^k \, 2^{2k+1} \, \omega_n^{(2k+1)} / (2k+1)\,! \\ \omega_k^{(m)} = \sum_{j=0}^{k} \omega_j^{(m-1)} N_{k-j} \\ \omega_j^{(1)} = N_j \\ N_j = \varphi_j - \beta \cdot \delta_{j0} \end{cases}$$

Moreover, the expansion coefficients of eq. (3) must satisfy the recurrence eq.:

$$(6) \qquad d^2 \varphi_k / d\eta^2 + q_{k-1} = 0$$

where $q_{-1} \equiv 0$.

By taking into account the boundary conditions, we may deduce obviously $\varphi_o(\eta) = 0$, and furthermore:(••)

(•) The Poincaré's theorem[13] insures that the expansion (3) always converges, and if $\Xi \leqslant 1$ also; but in this case the convergence radius results too large.

(••) For example, $\beta \sim -10^{\circ}$ gives $\varphi_1(0) \sim 4 \cdot 10^{-2}$, $\varphi_2(0) \sim 7 \cdot 10^{-3}$ and $\varphi_3(0) \sim \sim 10^{-3}$: hence the expansion may be stopped at the third term, if $\Xi > 1$.

$$(7)\quad \begin{cases} \varphi_1(\eta) = (1/2)\sin 2\beta\,(\eta^2 - 1/4) \\ \varphi_2(\eta) = -(1/8)\sin 4\beta\,[(1/3)(\eta^4 - 1/16) - (1/2)(\eta^2 - 1/4)] \end{cases}$$

Now, it is well known that, for non dissipative homeotropic NLC cell, it results[11]:

$$(8)\qquad C_{\|}/C = \sum_{n=0}^{\infty} e^n I_{2n}$$

By eq. (3) we have

$$(9)\qquad \sin\varphi = \sum_{k=0}^{\infty} Q_k \cdot \Xi^{-2k}$$

where the expansion coefficients are defined by:

$$(10)\quad \begin{cases} Q_k = \sum_{n=0}^{\infty} (-1)^n\, \Omega_k^{(2n+1)} / (2n+1)! \\ \Omega_k^{(m)} = \sum_{j=0}^{k} \Omega_j^{(m-1)}\, \varphi_{k-j} \\ \Omega_k^{(1)} = \varphi_k \end{cases}$$

Hence we can deduce:

$$(11)\qquad C_{\|}/C = \sum_{k=0}^{\infty} \langle G_k \rangle \cdot \Xi^{-2k}$$

where

$$(12)\quad \begin{cases} \langle G_k \rangle = \sum_{n=0}^{\infty} e^n \langle \Gamma_k^{(2n)} \rangle \\ \sin^{2n}\varphi = \sum_{k=0}^{\infty} \Gamma_k^{(2n)} \cdot \Xi^{-2k} \\ \Gamma_k^{(m)} = \sum_{j=0}^{k} \Gamma_j^{(m-1)}\, Q_{k-j} \end{cases}$$

The strong anchoring(■) gets the surprisingly simple relations:

$$(13)\quad \begin{cases} < G_0\ (\eta) > \ = 1 \\ < G_1\ (\eta) > \ = 0 \\ < G_2\ (\eta) > \ = e < \varphi_1^2\ (\eta) > = e \sin^2 2\beta \ / \ 120 \\ < G_3\ (\eta) > \ = 2e < \varphi_1(\eta)\, \varphi_2(\eta) > \sim e \sin^2 2\beta \cos 2\beta \ / \ 300 \end{cases}$$

and finally

$$(14)\qquad C_{\|} \ / \ C = 1 + (e/120) \sin^2 2\beta \cdot \Xi^{-4} \ [1 + (3/5) \cos 2\beta \cdot \Xi^{-2}\,] + \\ + O(\Xi^{-8})$$

If $\Xi > 1$, the percentage variation of the cell capacitance:

$$(15)\qquad \delta C \ / \ C_{\|} \sim -(e/120) \sin^2 2\beta \cdot \Xi^{-4}$$

is foreseen to be depending on the square US intensity(■■), for low intensity of a US burst obliquely incident on the homeotropic NLC cell. Eq. (15) suggests a very simple method for testing the validity of Dion's theory: the measurements are in progress, and will be discussed in another paper.

(■) In the case of weak anchoring, $< G_k\ (\eta) >$ results a polynomial function of k-th order in e, with the coefficients depending on β and φ ($\pm$ 1/2).

(■■) If $\beta = -10^{\circ}$ and $\Xi^2 \sim 60$, eq. (15) differs from eq. (14) by a term less than 1%.

LIST OF SYMBOLS

NLC	=	Nematic Liquid Crystal(-s)
US	=	Ultrasound(-s)
MCB	=	Magnetically Controlled Birefringence
ECB	=	Electrically Controlled Birefringence
ACB	=	Acoustically Controlled Birefringence
ξ	=	coherence length
K	=	elastic constant
v	=	US velocity in the NLC
h	=	NLC film thickness
I	=	US intensity incident on the NLC film
α_a	=	anisotropy of the US attenuation in the NLC (Nep/cm)
η	=	reduced abscissa normal to the cell plates $(-1/2 \leqslant \eta \leqslant 1/2)$
$\varphi(\eta)$	=	local tilt angle with respect to the normal to the cell plates
β	=	angle of the US wave vector in the NLC with respect to the normal to the cell plates
Ξ	=	ξ/h
$C_{\parallel}$	=	capacity of the homeotropic NLC cell
δC	=	capacity variation due to the US reorienting effect
$\varphi_k(\eta)$	=	expansion coefficients of $\varphi(\eta)$
$q_n(\eta)$	=	expansion coefficients of $\sin 2(\varphi - \beta)$
$\omega_j^{(m)}(\eta)$, $N_j(\eta)$	=	recurrence variables

C	$= C_{\parallel} + \delta C$
e	$= \epsilon_a / \epsilon_{\parallel}$
I_{2n}	$= \int_{-1/2}^{1/2} \sin^{2n} \varphi \cdot d\eta$
$Q_k(\eta)$	= expansion coefficients of $\sin \varphi$
$\Omega_j^{(m)}(\eta)$	= recurrence variable
$< G_k(\eta) >$	= expansion coefficients of $C_{\parallel} / C$
$\Gamma_k^{(2n)}(\eta)$	= expansion coefficients of $\sin^{2n} \varphi$

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